Characterization of stability of contractions

Attila Szalai

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Based on a joint work with László Kérchy.

Hilbert Function Spaces *May 23, 2013, Gargnano*

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 $\begin{array}{l} \mathcal{H} \mbox{ complex, separable Hilbert space, } \dim \mathcal{H} = \aleph_0. \\ \mathcal{L}(\mathcal{H}) \mbox{ bounded, linear operators on } \mathcal{H}. \\ \mathcal{T} \in \mathcal{L}(\mathcal{H}), \ ||\mathcal{T}|| \leq 1 \end{array}$

- $T = T_1 \oplus T_2$
 - T_1 is c.n.u.: $\not\exists \mathcal{M} \in \operatorname{Lat} T_1$: $T_1 | \mathcal{M}$ is unitary
 - T₂ is unitary
 - *T* is absolutely continuous if *T*₂ is a.c.,
 - i.e. if the spectral measure of T_2 is a.c. with respect to Lebesgue measure



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 $T \in \mathcal{L}(\mathcal{H})$ a.c. contraction

Minimal unitary dilation $U \in \mathcal{L}(\mathcal{G})$ of T: (i) $\mathcal{H} \subset \mathcal{G}, \bigvee_{n=-\infty}^{\infty} U^n \mathcal{H} = \mathcal{G},$ (ii) $T^n = P_{\mathcal{H}} U^n | \mathcal{H} \forall n \in \mathbb{Z}_+.$

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Image: A matrix



 $T \in \mathcal{L}(\mathcal{H})$ a.c. contraction

U a.c. unitary operator

 $\exists! \Phi_U : L^{\infty} \to \mathcal{L}(\mathcal{G}), f \mapsto f(U) \text{ weak-* continuous, contractive,} unital algebra-homomorphism, such that <math>\chi(U) = U$ (where $\chi(\zeta) = \zeta \ \forall \zeta \in \mathbb{T}$)

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Introduction $\circ \bullet \circ \circ \circ$ Hardy space H^{∞} Contractions

$$\mathcal{H}^{\infty} = \left\{ f \in L^{\infty} : \ \hat{f}(-n) = \int_{\mathbb{T}} f \chi^n \, \mathrm{d}\mu = 0 \,\,\forall n \in \mathbb{N} \right\}$$

Hardy space, weak-*-closed subalgebra of L^{∞}

 $f \in H^{\infty} \implies F(z) = \int_{\mathbb{T}} \frac{1-|z|^2}{|1-\overline{\zeta}z|^2} f(\zeta) d\mu(\zeta) \ (z \in \mathbb{D})$ bounded analytic function on \mathbb{D} .

 $F : \mathbb{D} \to \mathbb{C}$ bounded analytic $\Longrightarrow f \in H^{\infty}$, where $f(\zeta) = \lim_{r \to 1^{-}} F(r\zeta)$ for a.e. $\zeta \in \mathbb{T}$.

 $f \equiv F$

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Image: A matrix

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- weak-* continuous
- contractive: $||f(T)|| \le ||f||_{\infty}$
- unital algebra-homomorphism
- $\chi(T) = T$

Uniquely determined:

Sz.-Nagy–Foias functional calculus for T.

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Definition $T \in C_0$, if ker $\Phi_T \neq \{0\}$

Then $\exists ! m_T \in H^{\infty}$ inner function, such that ker $\Phi_T = m_T H^{\infty}$. m_T minimal function of T.

Example.
$$\vartheta \in H^{\infty}$$
 inner: $|\vartheta(\zeta)| = 1$ for a.e. $\zeta \in \mathbb{T}$
 $H^2 = \left\{ f \in L^2 : \hat{f}(-n) = 0 \ \forall n \in \mathbb{N} \right\}$ - analytic subspace of $L^2(\mu)$

$$\begin{split} \mathcal{H}(\vartheta) &= H^2 \ominus \vartheta H^2 \\ \mathcal{S}(\vartheta) \in \mathcal{L}(\mathcal{H}(\vartheta)), \, \mathcal{S}(\vartheta)f = \mathcal{P}_{\mathcal{H}(\vartheta)}(\chi f) \end{split}$$

$$S(\vartheta) \in C_0$$
 and $m_{S(\vartheta)} = \vartheta$

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Classes of contractions, stability

$T \in C_{0}$: $T^n \rightarrow 0$ (SOT), that is $T^n x \rightarrow 0 \ \forall x \in \mathcal{H}$

$T \in C_{1.}: T^n x e 0$ for every $x \in \mathcal{H}$

Dritschel: $h_n(T) \rightarrow 0$ (SOT) ??? $T^n \rightarrow 0$ (SOT)

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$T \in C_{0}$: $T^n \rightarrow 0$ (SOT), that is $T^n x \rightarrow 0 \ \forall x \in \mathcal{H}$

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Definition

A sequence of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ is a *test sequence of stability for a.c. contractions* if for every a.c. contraction *T* the condition $T^n \to 0$ (SOT) holds exactly when $h_n(T) \to 0$ (SOT).

Theorem (2012 Kérchy, Sz.)

A sequence of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ is a test sequence of stability for a.c. contractions if and only if

(i)
$$\lim_{n\to\infty} h_n(z) = 0$$
 for all $z \in \mathbb{D}$,

(ii) sup
$$\{||h_n||_{\infty} : n \in \mathbb{N}\} < \infty$$
,

(iii) lim sup_{n→∞} ||χ_αh_n||₂ > 0 for every Borel set α ⊂ T of positive measure.

 $(\chi_{\alpha}$ is the characteristic function of α .)

200

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Test sequences of stability

- The conditions (i) and (ii) together mean that {*h_n*}[∞]_{n=1} converges to zero in the weak-* topology.
- Examples:
 - $h_n = u^n$, where *u* is a non-constant inner.

•
$$h_n = \chi^{n+1} - \chi^n$$
.

• $T \in C_0 \implies \exists \vartheta \text{ inner},$ $\vartheta(T) = 0 \implies \vartheta^n(T) = 0 \rightarrow 0 (SOT) \implies T \in C_0.$

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Let us consider the unilateral shift of multiplicity one: $S \in \mathcal{L}(H^2)$, $Sf = \chi f$, $(\chi(z) = z)$.

$$S^{*n}
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For the Cauchy kernel $k_{\lambda}(z) = \frac{1}{1-\overline{\lambda}z}$:

$$S^*k_{\lambda} = \overline{\lambda}k_{\lambda} \Rightarrow h_n(S^*)k_{\lambda} = h_n(\overline{\lambda})k_{\lambda} \Rightarrow h_n(\overline{\lambda}) \to 0 \ \forall \ \lambda \in \mathbb{D},$$

that is (i) holds.

$$||h||_{\infty} \geq ||h(S^*)|| \geq \frac{||h(S^*)k_{\lambda}||_2}{||k_{\lambda}||_2} = |h(\overline{\lambda})|$$

for all $\lambda \in \mathbb{D}$. Thus the Banach–Steinhaus theorem shows that $\sup_n ||h_n||_{\infty} = \sup_n ||h_n(S^*)|| < \infty$, that is (ii) holds.

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for all $\lambda \in \mathbb{D}$. Thus the Banach–Steinhaus theorem shows that $\sup_n ||h_n||_{\infty} = \sup_n ||h_n(S^*)|| < \infty$, that is (ii) holds.

Let $\alpha \subset \mathbb{T}$, $m(\alpha) > 0$, $L^2(\alpha) = \chi_{\alpha}L^2(\mathbb{T})$. Then $M_{\alpha} \in \mathcal{L}(L^2(\alpha))$, $M_{\alpha}g = \chi g$ is an a.c. unitary operator, hence

 $M^n_{\alpha} \not\rightarrow 0 \text{ (SOT)} \Rightarrow h_n(M_{\alpha}) \not\rightarrow 0 \text{ (SOT)}.$

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$T \in C_{0.} \Rightarrow T \cong S_{\infty}^* | \mathcal{M}$ (Rota, Foias). $(S_{\infty}^* = S^* \oplus S^* \oplus \ldots).$

• $h_n(S^*)k_{\lambda} = h_n(\overline{\lambda})k_{\lambda} \to 0$ for all $\lambda \in \mathbb{D}$ by (i),

•
$$\vee \{k_{\lambda} : \lambda \in \mathbb{D}\} = H^2$$
,

- $\{h_n(S^*)\}_{n=1}^{\infty}$ is bounded by (ii).
- $\Rightarrow h_n(S^*) \rightarrow 0, h_n(S^*_{\infty}) \rightarrow 0 \text{ and } h_n(T) \rightarrow 0 \text{ (SOT)}.$



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• $h_{n}(S^{*})k_{\lambda} = h_{n}(\overline{\lambda})k_{\lambda} \to 0 \text{ for all } \lambda \in \mathbb{D} \text{ by (i),}$
• $\forall \{k_{\lambda} : \lambda \in \mathbb{D}\} = H^{2},$
• $\{h_{n}(S^{*})\}_{n=1}^{\infty} \text{ is bounded by (ii).}$

 \Rightarrow $h_n(S^*) \rightarrow 0$, $h_n(S^*_{\infty}) \rightarrow 0$ and $h_n(T) \rightarrow 0$ (SOT).

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$T \notin C_{0.}$ can be written in the form

$$T = \begin{bmatrix} T_0 & * \\ 0 & T_1 \end{bmatrix} \in \mathcal{L}(\mathcal{H}) = \mathcal{L}(\mathcal{H}_0 \oplus \mathcal{H}_1)$$

where $\mathcal{H}_1 \neq \{0\}, \ T_0 \in C_{0\cdot}, \ T_1 \in C_{1\cdot}$. Therefore

$$h(T) = \begin{bmatrix} h(T_0) & * \\ 0 & h(T_1) \end{bmatrix}$$

for all $h \in H^{\infty}$. Assume to the contrary that $h_n(T) \to 0$ (SOT). This implies that $h_n(T_1) \to 0$ (SOT). This leads to a contradiction via the concept unitary asymptote.

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Characterization of stability of contractions

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Proposition

Let $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$. Then $h_n(T) \to 0$ (SOT) for all $T \in C_0$. if and only if $\{h_n\}_{n=1}^{\infty}$ satisfies the conditions (i) and (ii).

Proposition

Let $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$. Then $h_n(T) \to 0$ (SOT) for every a.c. contraction T exactly when $\{h_n\}_{n=1}^{\infty}$ is a bounded sequence and $\lim_{n\to\infty} ||h_n||_2 = 0$.

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Image: A matrix

duction Cor

 $T \in \mathcal{L}(\mathcal{H})$ is polynomially bounded if $||p(T)|| \leq K_T ||p||_{\infty}$.

 Φ_{T,0} : P(T) → L(H), p → p(T) is a bounded algebra-homomorphism which extends continuously to the disc algebra:

•
$$\Phi_{T,1}: A \to \mathcal{L}(\mathcal{H}), f \mapsto f(T).$$

Mlak introduced and studied elementary measures of polynomially bounded operators. If *T* is a polynomially bounded operator then uniquely exist $\mathcal{H}_a, \mathcal{H}_s \in \text{Hlat } T, \mathcal{H} = \mathcal{H}_a \dotplus \mathcal{H}_s$ such that $T_a = T | \mathcal{H}_a$ is absolutely continuous and $T_s = T | \mathcal{H}_s$ is singular.

• *T* admits an H^{∞} -functional calculus exactly when *T* is a.c. polynomially bounded operator.

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A sequence of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ is a *test sequence of stability for a.c. polynomially bounded operators* if for every a.c. polynomially bounded operator $T \in \mathcal{L}(\mathcal{H})$ the property $T^n \to 0$ (SOT) holds exactly when $h_n(T) \to 0$ (SOT).

Theorem

A sequence of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ is a test sequence of stability for a.c. polynomially bounded operators if and only if $\{h_n\}_{n=1}^{\infty}$ converges to zero exclusively on \mathbb{D} .

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Attila Szalai

• T is a singular polynomially bounded operator if and only if T is similar to a singular unitary operator.

Proposition

Let $\{h_n\}_{n=1}^{\infty} \subset A$ be a bounded sequence. Then $h_n(T) \to 0$ (SOT) for every singular polynomially bounded operator T if and only if $\lim_{n\to\infty} h_n(\zeta) = 0$ for every $\zeta \in \mathbb{T}$. In that case $h_n(T) \to 0$ (SOT) for every polynomially bounded operator T.



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