Characterization of stability of contractions

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Based on a joint work with László Kérchy.

Hilbert Function Spaces
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$\mathcal{H}$ complex, separable Hilbert space, $\dim \mathcal{H} = \aleph_0$.
$\mathcal{L}(\mathcal{H})$ bounded, linear operators on $\mathcal{H}$.
$T \in \mathcal{L}(\mathcal{H}),\ ||T|| \leq 1$

- $T = T_1 \oplus T_2$
  - T_1 is c.n.u.: $\nexists\, \mathcal{M} \in \mathrm{Lat}\, T_1$: $T_1|\mathcal{M}$ is unitary
  - T_2 is unitary
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$T \in \mathcal{L}(\mathcal{H})$ a.c. contraction

Minimal unitary dilation $U \in \mathcal{L}(\mathcal{G})$ of T:

(i) $\mathcal{H} \subset \mathcal{G}$, $\bigvee_{n=-\infty}^{\infty} U^n\mathcal{H} = \mathcal{G}$,

(ii) $T^n = P_{\mathcal{H}} U^n|\mathcal{H}\ \forall n \in \mathbb{Z}_+$.

U a.c. unitary operator

$\exists!\ \Phi_U : L^\infty \to \mathcal{L}(\mathcal{G}), f \mapsto f(U)$ weak-* continuous, contractive, unital algebra-homomorphism, such that $\chi(U) = U$ (where $\chi(\zeta) = \zeta\ \forall \zeta \in \mathbb{T}$)

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$$H^\infty = \left\{ f \in L^\infty : \hat{f}(-n) = \int_{\mathbb{T}} f \chi^n \, \mathrm{d}\mu = 0 \ \forall n \in \mathbb{N} \right\}$$

Hardy space, weak-*-closed subalgebra of L^∞

$f \in H^\infty \implies F(z) = \int_{\mathbb{T}} \frac{1-|z|^2}{|1-\bar{\zeta}z|^2} f(\zeta)\mathrm{d}\mu(\zeta) \ \ (z \in \mathbb{D})$
bounded analytic function on $\mathbb{D}$.

$F : \mathbb{D} \to \mathbb{C}$ bounded analytic $\implies f \in H^\infty$, where
$f(\zeta) = \lim_{r\to 1-} F(r\zeta)$ for a.e. $\zeta \in \mathbb{T}$.

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Definition

$T \in C_0$, if $\ker \Phi_T \neq \{0\}$

Then $\exists!\ m_T \in H^\infty$ inner function, such that $\ker \Phi_T = m_T H^\infty$.
m_T minimal function of T.

Example.
$\vartheta \in H^\infty$ inner: $|\vartheta(\zeta)| = 1$ for a.e. $\zeta \in \mathbb{T}$
$H^2 = \left\{ f \in L^2 : \hat{f}(-n) = 0\ \forall n \in \mathbb{N} \right\}$ - analytic subspace of $L^2(\mu)$

$\mathcal{H}(\vartheta) = H^2 \ominus \vartheta H^2$
$S(\vartheta) \in \mathcal{L}(\mathcal{H}(\vartheta))$, $S(\vartheta) f = P_{\mathcal{H}(\vartheta)}(\chi f)$

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$T \in C_{0\cdot}$: $T^n \to 0$ (SOT), that is $T^n x \to 0$ $\forall x \in \mathcal{H}$

$T \in C_{1\cdot}$: $T^n x \not\to 0$ for every $x \in \mathcal{H}$

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A sequence of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ is a *test sequence of stability for a.c. contractions* if for every a.c. contraction T the condition $T^n \to 0$ (SOT) holds exactly when $h_n(T) \to 0$ (SOT).

Theorem (2012 Kérchy, Sz.)

A sequence of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ is a test sequence of stability for a.c. contractions if and only if

(i) $\lim_{n\to\infty} h_n(z) = 0$ *for all* $z \in \mathbb{D}$,

(ii) $\sup\{\|h_n\|_\infty : n \in \mathbb{N}\} < \infty$,

(iii) $\limsup_{n\to\infty} \|\chi_\alpha h_n\|_2 > 0$ *for every Borel set* $\alpha \subset \mathbb{T}$ *of positive measure.*
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- The conditions (i) and (ii) together mean that $\{h_n\}_{n=1}^{\infty}$ converges to zero in the weak-$*$ topology.
- Examples:
  - $h_n = u^n$, where u is a non-constant inner.
  - $h_n = \chi^{n+1} - \chi^n$.
- $T \in C_0 \implies \exists \vartheta$ inner, $\vartheta(T) = 0 \implies \vartheta^n(T) = 0 \to 0\ (SOT) \implies T \in C_{0\cdot}$.

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Let us consider the unilateral shift of multiplicity one:
$S \in \mathcal{L}\left(H^2\right)$, $Sf = \chi f$, $(\chi(z) = z)$.

$$S^{*n} \to 0 \text{ (SOT)} \;\Rightarrow\; h_n(S^*) \to 0 \text{ (SOT)}$$

For the Cauchy kernel $k_\lambda(z) = \frac{1}{1-\overline{\lambda}z}$:

$$S^* k_\lambda = \overline{\lambda} k_\lambda \;\Rightarrow\; h_n(S^*)k_\lambda = h_n(\overline{\lambda})k_\lambda \;\Rightarrow\; h_n(\overline{\lambda}) \to 0 \;\; \forall\, \lambda \in \mathbb{D},$$

that is (i) holds.

$$||h||_\infty \geq ||h(S^*)|| \geq \frac{||h(S^*)k_\lambda||_2}{||k_\lambda||_2} = |h(\overline{\lambda})|$$

for all $\lambda \in \mathbb{D}$. Thus the Banach–Steinhaus theorem shows that $\sup_n ||h_n||_\infty = \sup_n ||h_n(S^*)|| < \infty$, that is (ii) holds.

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Proof $\Rightarrow$

Let $\alpha \subset \mathbb{T}$, $m(\alpha) > 0$, $L^2(\alpha) = \chi_\alpha L^2(\mathbb{T})$.
Then $M_\alpha \in \mathcal{L}\left(L^2(\alpha)\right)$, $M_\alpha g = \chi g$ is an a.c. unitary operator, hence

$$M_\alpha^n \not\to 0 \text{ (SOT)} \;\Rightarrow\; h_n(M_\alpha) \not\to 0 \text{ (SOT)} .$$

Proof $\Leftarrow$

$T \in C_{0\cdot} \Rightarrow T \cong S^*_\infty|\mathcal{M}$ (Rota, Foias).
$(S^*_\infty = S^* \oplus S^* \oplus \ldots)$.

- $h_n(S^*)k_\lambda = h_n(\overline{\lambda})k_\lambda \to 0$ for all $\lambda \in \mathbb{D}$ by (i),
- $\vee\{k_\lambda : \lambda \in \mathbb{D}\} = H^2$,
- $\{h_n(S^*)\}_{n=1}^\infty$ is bounded by (ii).

$\Rightarrow h_n(S^*) \to 0$, $h_n(S^*_\infty) \to 0$ and $h_n(T) \to 0$ (SOT).

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$T \notin C_{0\cdot}$ can be written in the form

$$T = \begin{bmatrix} T_0 & * \\ 0 & T_1 \end{bmatrix} \in \mathcal{L}(\mathcal{H}) = \mathcal{L}(\mathcal{H}_0 \oplus \mathcal{H}_1)$$

where $\mathcal{H}_1 \neq \{0\},\ T_0 \in C_{0\cdot},\ T_1 \in C_{1\cdot}$. Therefore

$$h(T) = \begin{bmatrix} h(T_0) & * \\ 0 & h(T_1) \end{bmatrix}$$

for all $h \in H^\infty$. Assume to the contrary that $h_n(T) \to 0$ (SOT). This implies that $h_n(T_1) \to 0$ (SOT). This leads to a contradiction via the concept unitary asymptote.

Proposition

Let $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$. *Then* $h_n(T) \to 0$ *(SOT) for all* $T \in C_0$. *if and only if* $\{h_n\}_{n=1}^{\infty}$ *satisfies the conditions* (i) *and* (ii).

Proposition

Let $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$. *Then* $h_n(T) \to 0$ *(SOT) for every a.c. contraction T exactly when* $\{h_n\}_{n=1}^{\infty}$ *is a bounded sequence and* $\lim_{n\to\infty} ||h_n||_2 = 0$.

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$T \in \mathcal{L}(\mathcal{H})$ is polynomially bounded if $||p(T)|| \leq K_T||p||_\infty$.

- $\Phi_{T,0} : \mathcal{P}(\mathbb{T}) \to \mathcal{L}(\mathcal{H})$, $p \mapsto p(T)$ is a bounded algebra-homomorphism which extends continuously to the disc algebra:
- $\Phi_{T,1} : A \to \mathcal{L}(\mathcal{H})$, $f \mapsto f(T)$.

Mlak introduced and studied elementary measures of polynomially bounded operators . If T is a polynomially bounded operator then uniquely exist $\mathcal{H}_a, \mathcal{H}_s \in \mathrm{Hlat}\, T, \mathcal{H} = \mathcal{H}_a \dot{+} \mathcal{H}_s$ such that $T_a = T|\mathcal{H}_a$ is absolutely continuous and $T_s = T|\mathcal{H}_s$ is singular.

- T admits an H^∞-functional calculus exactly when T is a.c. polynomially bounded operator.

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Theorem

A sequence of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ is a test sequence of stability for a.c. polynomially bounded operators if and only if $\{h_n\}_{n=1}^{\infty}$ converges to zero exclusively on $\mathbb{D}$.

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- T is a singular polynomially bounded operator if and only if T is similar to a singular unitary operator.

Proposition

Let $\{h_n\}_{n=1}^{\infty} \subset A$ *be a bounded sequence. Then* $h_n(T) \to 0$ *(SOT) for every singular polynomially bounded operator T if and only if* $\lim_{n\to\infty} h_n(\zeta) = 0$ *for every* $\zeta \in \mathbb{T}$*. In that case* $h_n(T) \to 0$ *(SOT) for every polynomially bounded operator* T*.*

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