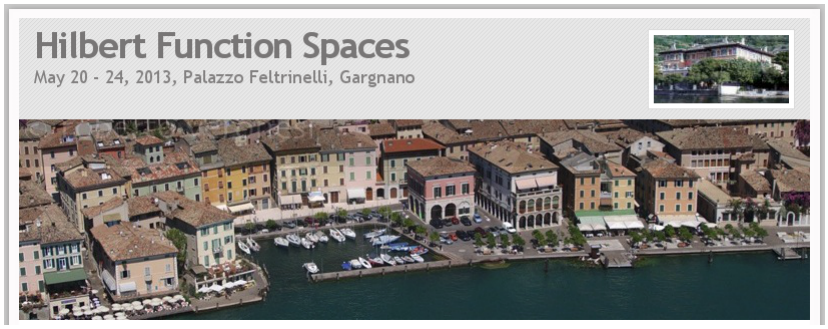

Titles and abstract

Jim Agler
University of California, San Diego

Non-commutative function theory

Alexandru Aleman
Lund University

On the Sarason conjecture for the Bergman space

Abstract. We give a characterization of those analytic functions f, g in the unit disc with the property that the Toeplitz product $T_f T_g^*$ is bounded on the Bergman space. We shall also discuss some non-analytic examples which show the connection between the Sarason-type condition and Carleson measures on the Dirichlet space. This is joint work with Sandra Pott and Maria Reguera.

Yurii Belov
Chebyshev laboratory, St Petersburg State University

Approximation of L^2 function on an interval by shifts and exponentials

Abstract. Let f be a function in $L^2[0,1]$ with support in $[0,a]$, $0 < a < 1$. Consider the family of shifts $f(x-t)$, $0 < t < 1-a$. In contrast to the classical Wiener theorem, this family is never complete in $L^2[0,1]$, and the orthogonal complement contains the exponentials corresponding to the zeros of the Fourier transform of f. The question posed by M. Carlsson and C. Sundberg is whether the system consisting of such exponentials and of the initial family of shifts is complete in $L^2[0,1]$. We show that the answer to this question is positive.

Kelly Bickel
Washington University in St Louis

Differentiating Matrix Functions

Abstract. Every real-valued function defined on $\mathbb{R}^2$ induces a matrix-valued function on the space of pairs of commuting self-adjoint *nimesn* matrices, denoted CS_n. We discuss results about the geometry of the space CS_n and show that a C^1 function always induces a matrix function that can be continuously differentiated along C^1 curves. With additional restrictions on the domain of the original function, these results extend to higher-order differentiation.

Aline Bonami
University of Orleans

Generalized Hardy spaces in R^n, products, paraproducts and div-curl lemma.

Abstract. I will first describe joint work with Sandrine Grellier et Luong Dang Ky and prove that the product of a function in H^1 and a function in BMO involves a new kind of Hardy space in R^n. Then I will give some properties of these Hardy spaces that have been developed by Luong Dang Ky and other authors.

Alexander Borichev
Aix-Marseille Université

Generalized Carleson-Newman inner functions

Abstract. The class of the Carleson-Newman Blaschke products B is determined by the condition

$$|B(z)| \asymp \rho(z, Z(B))^N, \qquad z \in \mathbb{D},$$

where $N = N(B) < \infty$, $Z(B)$ is the zero set of B, $\rho(z, w) = |z-w|/|1-z\bar{w}|$, and $\rho(z, E) = \inf_{w\in E} \rho(z, w)$. We discuss a natural generalization of this class motivated by interesting operator theory applications.

Marcus Carlsson
Lund University

On AAK-theory for Hankel operators on the Dirichlet space

Abstract. One version of the AAK-theorem for Hankel operators on the Hardy space says that one can find a shift-invariant subspace of codimension k, such that the operator restricted to this subspace has norm equal to the corresponding singular number. Moreover the subspace in question is defined by k zeroes that can be explicitly found by looking at the corresponding singular vector. We show that the same holds true in the Dirichlet space, and discuss similar statements for other spaces where the shift is an expansive operator.

Sameer Chavan
IIT Kanpur

Spherically Balanced Hilbert Spaces of Formal Power Series in Several Variable

Abstract. Consider the complex Hilbert space $H^2(\beta)$ of formal power series in the variables $z_1, \cdots, z_m$ endowed with the norm $\|f\|^2_{H^2(\beta)} := \sum_{n\in\mathbb{Z}^m_+} |\hat{f}(n)|^2\beta_n^2$ $(f \in H^2(\beta))$, where $\beta_0 = 1$. Motivated by the theory of spherical Cauchy dual [1], we study the spherically balanced spaces, that is, spaces $H^2(\beta)$ for which the multi-sequence $\{\beta_n\}_{n\in\mathbb{Z}^m_+}$ satisfies

$$\sum_{k=1}^m \beta^2_{n+\varepsilon_i+\varepsilon_k}/\beta^2_{n+\varepsilon_i} = \sum_{k=1}^m \beta^2_{n+\varepsilon_j+\varepsilon_k}/\beta^2_{n+\varepsilon_j} \ (n \in \mathbb{Z}^m_+,\ i, j = 1, \cdots, m),$$

where ε_j is the m-tuple with 1 in the jth place and zeros elsewhere. The space $H^2(\beta)$ is spherically balanced if and only if there exist a Reinhardt measure μ supported on the unit sphere $\partial\mathbb{B}$ and a Hilbert space $H^2(\gamma)$ of formal power series in the variable t such that

$$\|f\|^2_{H^2(\beta)} = \int_{\partial\mathbb{B}} \|f_z\|^2_{H^2(\gamma)}\, d\mu(z) \ (f \in H^2(\beta)),$$

where $f \in H^2(\beta)$ and $f_z(t) = f(tz)$ is a formal power series in the variable t. As an application, we obtain a result concerning cyclic vectors of joint complete hyperexpansive m-tuples. This generalizes [2, Theorem 5.3].

This is a joint work with Surjit Kumar.

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[2] J. Gleason and S. Richter, *m-Isometric commuting tuples of operators on a Hilbert space*, Inte. equ. oper. theory, **56** (2006), 181-196.

Carl Cowen
Indiana University - Purdue University Indianapolis

Rota's Universal Operators and Invariant Subspaces in Hilbert Spaces

Abstract. Rota showed, in 1960, that there are operators T that provide models for every bounded linear operator on a separable infinite dimensional Hilbert space, in the sense that given an operator A on such a Hilbert space, there is $\lambda \neq 0$ and an invariant subspace M for T such that the restriction of T to M is similar to λA. In 1969, Caradus provided a practical condition for identifying such universal operators. In this talk, we will use the Caradus theorem to exhibit a new example of a universal operator and show how it can be used to provide information about invariant subspaces for Hilbert space operators. Of course, Toeplitz operators and composition operators on the Hardy space $H^2(\mathbb{D})$ will play a role!

This talk describes work in collaboration with Eva Gallardo-Gutiérrez, Universidad Complutense de Madrid e IUMA, done there this year during the speaker's sabbatical.

Michael Dritschel
Newcastle University

Realizations via Preorderings

Abstract. We extend Jim Agler's notion of a function space defined in terms of test functions to include products, in analogy with the practice in real algebraic geometry. In this context we prove a realization theorem for functions in a generalization of the Schur-Agler class and which includes the Schur class on the polydisk as a special case. In the setting of so-called ample preorderings, we can further strengthen the realization theorem, allowing for applications to, for example, Pick type interpolation problems. A restricted class of representations called (weakly continuous) Brehmer representations are shown to be completely contractive, implying a form of rational dilation. Time permitting, we discuss further applications as well.

Miroslav Englis
Akademie Věd, Praha

Toeplitz and Hankel operators on weighted Fock spaces

Abstract. We give criteria for the membership of Toeplitz operators and of products of Hankel operators, with symbols of a certain type, in Schatten ideals and in the Dixmier class, and formulas for their Dixmier trace, on a variety of weighted Segal-Bargmann-Fock spaces on the complex plane. (Joint work with H. Bommier and E.-H. Youssfi, Marseille.)

Daniel Estévez
Universidad Automa de Madrid

Explicit traces of functions of Sobolev spaces and quasi-optimal linear interpolators

Abstract. iven a strictly increasing sequence $\Lambda \subset \mathbb{R}$, we consider the trace spaces $L^r_p(\mathbb{R})_\Lambda$,$W^r_p(\mathbb{R})_\Lambda$. These are the sequence spaces obtained by restricting to Λ the functions on the homogeneous Sobolev spaces $L^r_p(\mathbb{R})$ and non-homogeneous Sobolev spaces $W^r_p(\mathbb{R})$. The problem under study is to give an explicit characterization of the trace spaces $\mathbb{X}_\Lambda$, where $\mathbb{X} = L^r_p(\mathbb{R})$ or $W^r_p(\mathbb{R})$. In particular, we are interested in a simple formula for an equivalent norm in $\mathbb{X}_\Lambda$. We would also like to find a linear bounded extension

operator $T : \mathbb{X}_\Lambda o \mathbb{X}$ which is computationally simple. We solve these problems for $r = 1, 2$ by giving a simple and explicit formula for an equivalent norm in the trace space and constructing a linear bounded extension operator which is given by some kind of polynomial spline interpolation. We also give a general result that relates interpolation on $L_p^r(\mathbb{R})$ and $W_p^r(\mathbb{R})$ for any $r \geq 1$.

Quanlei Fang
CUNY-BCC

A local inequality for Hankel operators on the sphere

Abstract. Let $H^2(S)$ be the Hardy space on the unit sphere S in $\mathbf{C}^n$. We establish a local inequality for Hankel operators $H_f = (1 - P)M_f|H^2(S)$. As an application of this local inequality, we characterize the membership of H_f in the Lorentz-like ideal $\mathcal{C}_p^+$, $2n < p < \infty$. This is a joint work with Jingbo Xia.

Tim Ferguson
Vanderbilt University

Self-commutators of Toeplitz operators and isoperimetric inequalities

Abstract. D. Khavinson (1985) obtained lower bounds for the norms of Toeplitz operators with analytic symbol acting on the Smirnov space of a domain. When combined with Putnam's inequality for hyponormal operators, Khavinson's result implies the classical isoperimetric inequality. We consider self-commutators of Toeplitz operators with analytic symbol acting on the Bergman space of a domain, and obtain lower bounds that reflect the geometry of the domain. When combined with Putnam's inequality our lower bounds give rise to the Faber-Krahn inequality for the fundamental frequency of a domain and the Saint-Venant inequality for the torsional rigidity (but with non-sharp constants). We conjecture an improved version of Putnam's inequality within this restricted setting. This is joint work with Steve Bell and Erik Lundberg.

György Pál Gehér
Bolyai Institute, University of Szeged

Cyclic Properties and Isometric Asymptotes of Tree-shift Operators

Abstract. We investigate some properties of a new class of operators called tree-shift operators introduced recently in [JJS]. This class is a generalization of the so called weighted bilateral, unilateral and backward shift operators.

Let $\mathcal{T} = (V, E)$ be a *directed tree*, where V and E is the set of (generally infinitely many) *vertices* and *(directed) edges*, respectively. A vertex v is called a *child* of $u \in V$ if $(u, v) \in E$. The set of all children of u is denoted by $\mathrm{Chi}(u)$.

Let $\underline{\lambda} = \{\lambda_v \colon v \in V^\circ\} \subseteq \mathbb{C}$ be a set of *weights* such that

$$\sup\left\{\sqrt{\sum_{v\in\mathrm{Chi}(u)} |\lambda_v|^2} \colon u \in V\right\} < \infty.$$

Then the *tree-shift operator* is the operator defined by

$$S_{\underline{\lambda}} \colon \ell^2(V) \to \ell^2(V), \quad e_u \mapsto \sum_{v\in\mathrm{Chi}(u)} \lambda_v e_v,$$

which is bounded by the norm $\|S_{\underline{\lambda}}\| = \sup\left\{\sqrt{\sum_{v\in\mathrm{Chi}(u)} |\lambda_v|^2} \colon u \in V\right\}$.

Let $\mathcal{H}$ is a complex Hilbert space and $\mathcal{B}(\mathcal{H})$ is the C*-algebra of bounded linear operators on it. If $T \in \mathcal{B}(\mathcal{H})$ is a contraction (i.e. $\|T\| \leq 1$), then the sequences $\{T^{*n}T^n\}_{n=1}^\infty$ and $\{T^nT^{*n}\}_{n=1}^\infty$ of positive operators are decreasing, so they have unique limits in the strong operator topology (SOT):

$$A = A_T = \lim_{n\to\infty} T^{*n}T^n \quad \text{and} \quad A_* = A_{T^*} = \lim_{n\to\infty} T^nT^{*n}.$$

The operator A is the *asymptotic limit* of T and A_* is the *asymptotic limit* of the adjoint T^*.

We recall that $A_T^{1/2}$ acts as an intertwining mapping in a canonical realization of the so called *unitary* and *isometric asymptote* of the contraction T.

In this talk we describe the asymptotic limit of a tree-shift contraction and of the adjoint. Then we deal with the isometric and unitary asymptotes of these contractions. After that we investigate cyclic properties and give some similarity results. Some new properties of usual shift operators will be also introduced.

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This research was realized in the frames of TÁMOP 4.2.4. A/2-11-1-2012-0001 "National Excellence Program - Elaborating and operating an inland student and researcher personal support system". The project was subsidized by the European Union and co-financed by the European Social Fund.

Pratibha G. Ghatage
Cleveland State University of Ohio

Carleson measures and closed-range composition operators on Bergman and Dirichlet spaces

Abstract. For an analytic self-map φ of $\{z : |z| < 1\}$, we give a new necessary and sufficient condition for the composition operator C_φ to be bounded below on the classical Bergman space in terms of areas of inverse image of a Borel subset of $\mathbb{D}$. We use a condition developed in the process of the proof to give an example of an outer function which gives rise to a composition operator which is bounded below on the

Bergman space. We give a new condition which we is necessary and sufficient for a bounded composition operator to be bounded below on the Dirichlet space. We also relate it to an appropriate Carleson measure.

Gokhan Gogus
Sabanci University

Poletsky-Stessin-Hardy spaces in the plane

Abstract. We give two characterizations of the Poletsky-Stessin-Hardy spaces in the complex plane: First we completely describe functions in these spaces by having a harmonic majorant with a certain growth condition. Second we describe these functions in terms of their boundary values as a weighted subclass of the usual L^p class with respect to the arclength measure on the boundary. We extend the classical result of Beurling which describes the invariant subspaces of the shift operator. Additionally we provide non-trivial examples. Joint work with M. A. Alan.

Ugur Gul
Hacettepe University

Parabolic Non-Automorphism Induced Toeplitz-Composition C*-Algebras with Piece-wise Quasi-Continuous symbols

Abstract. In this talk we consider the C*-algebra $C(\{C\varphi\} \cup T(PQC(T)))/K(H^2)$ generated by Toeplitz operators with piece-wise quasi-continuous symbols and a composition operator induced by a parabolic linear fractional non-automorphism symbol modulo compact operators on the Hilbert-Hardy space H^2 . This C*-algebra is commutative. We characterize its maximal ideal space. We apply our results to the question of determining the essential spectra of linear combinations of a class of composition operators and Toeplitz operators.

Joshua Isralowitz
SUNY Albany

Compactness of operators on Generalized Fock Spaces

Abstract. In this talk, we discuss the question of when the Berezin transform characterizes the compactness of a bounded operator on a wide class of weighted Fock spaces. Furthermore, we discuss the question of whether all compact operators on these weighted Fock spaces are necessarily in the Toeplitz algebra.

Michael Jury
University of Florida

Function spaces on varieties

Abstract. This talk will survey some recent results on the function-theoretic operator theory (such as Nevanlinna-Pick interpolation and the rational dilation problem) on distinguished varieties in the bidisk and associated constrained function algebras. This is joint work with Michael Dritschel, Greg Knese, and Scott McCullough.

Turgay Kaptanoglu
Bilkent University

Harmonic Besov Hilbert Spaces on the Real Unit Ball

Abstract. We consider the harmonic Besov Hilbert spaces b_q^2 for all $q \in \mathbb{R}$ on the unit ball of $\mathbb{R}^n$. They generalize the weighted harmonic Bergman spaces which exist for $q > -1$. We define them by writing their reproducing kernels explicitly as weighted infinite sums of zonal harmonics. These kernels turn out te be radial fractional derivatives of the Poisson kernel for the ball. Alternately, we define the spaces b_q^2 using integral norms which place sufficiently high-order radial derivatives of their members in harmonic Bergman spaces. We show the equivalence of the two approaches. We discuss various properties of the kernels, characterizations of and connections among the spaces.

Greg Knese
University of Alabama

Polynomials with no zeros on a face of the bidisk

Abstract. We will present a sums of squares formula for polynomials with no zeros on a face of the bidisk which simultaneously generalizes a formula due to Cole and Wermer and a formula related to the Schur-Cohn method for counting the number of roots a one variable polynomial has in the unit disk. We will describe a proof inspired by work of Kummert. Joint work with J. Geronimo and P. Iliev.

Hyun Kwon
University of Alabama

Dirichlet contractions and similarity

Abstract. We show that the same similarity criterion that lets one decide when an n-hypercontraction is similar to the backward shift in certain function spaces can be used in dealing with Dirichlet contractions.

Wing Suet Li
Georgia Institute of Technology

On Horn inequalities for sums of Hermitian matrices and submodules

Abstract. Let $\alpha = (\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n \geq 0)$, $\beta = (\beta_1 \geq \beta_2 \geq \cdots \geq \beta_n \geq 0)$ and $\gamma = (\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n \geq 0)$ be three partitions of integers. It is well-known that such triple (α, β, γ) that satisfies the so-call Littlewood- Richardson rule describes the eigenvalues of the sum of n by n Hermitian matricies, i.e., Hermitian matrices $A,\ B,$ and$C$ such that $A+B = C$ with α (β and γ respectively) as the set of eigenvalues of A (B and C respectively). At the same time such triple also describes the Jordan decompositions of a nilpotent matrix T, T resticts to an invarint subspace M, and $T_{M^\perp}$ the compression of T onto the $M^\perp$. More precisely, T is similar to $J(\gamma) := J_{\gamma_1} \oplus J_{\gamma_2} \oplus \cdots J_{\gamma_n}$, $T|M$ is similar to $J(\alpha)$, and $T_{M^\perp}$ is similar to $J(\beta)$. (Here J_k denotes the Jordan cell of size k with 0 on the diagonal.) In addition, these partitions must also satisfy the Horn inequalities. In this talk I will explain the connections between these two seemily unrelated objects in matrix theory and why the same combinatorics works for both. This talk is based on the joint work with H. Bercovici and K. Dykema.

Constanze Liaw
Baylor University

Clark model in general situation

Abstract. We consider rank one unitary perturbations $U_\gamma = U + (\gamma - 1) < ., U^{-1}b > b$, $|\gamma| = 1$ of a unitary operator U by a cyclic vector b on a Hilbert space. The Clark operators Φ_γ are the family unitary operators that intertwine the spectral representation of U_γ and the corresponding perturbation of the model operator. It is well-known that inner characteristic functions, the adjoint operator Φ_γ^* is given by the normalized Cauchy transform – one of the most important objects in Clark theory. Allowing non-inner characteristic functions, we introduce a "universal" representation formula for the adjoint of the Clark operator on the free model space. Under imposing the Sz.-Nagy–Foiaś transcription (i.e. a particular choice of coordinates in the defect space) our formula provides a generalization of the normalized Cauchy transform. This work is joint with Sergei Treil.

Erik Lundberg
Purdue University

The Zero Set of a Random Fourier Polynomial on the Circle and Sphere

Abstract. We start with a question motivated by the fundamental theorem of algebra: How many zeros of a random polynomial are real? We discuss three Gaussian ensembles that lead to three different answers. Of these, we emphasize the analyst's model of choice which has the highest expected number of zeros (a fraction of the maximum) and reduces to a random trigonometric polynomial. The real section of the zero set of a polynomial in several variables is much more complicated. Hilbert's sixteenth problem asks to study the possible arrangements of the connected components, and is especially concerned with the case of many components. I will describe a probabilistic approach to studying the topology, volume, and arrangement of the zero set (in real projective space) for a Gaussian ensemble of homogeneous polynomials. In this case, the analyst's model of choice reduces to a random Fourier series of spherical harmonics. This is joint work with Antonio Lerario.

Zinaida Lykova
Newcastle University

Extremal holomorphic maps and the symmetrised bidisc

Abstract. Between any pair of domains we define the class of *n-extremal holomorphic maps*. This is a class of maps that generalises both finite Blaschke products and complex geodesics. We are mainly concerned with n-extremal holomorphic maps from the open unit disc $\mathbb{D}$ to the symmetrised bidisc $\Gamma = \{(z+w, zw) : |z| \leq 1, |w| \leq 1\}$. Initial interest in the function theory and complex geometry of Γ arose from its connection with control engineering, but subsequently it is the implications for the theory invariant distances that has proved most significant. The original engineering problem leads to the finite interpolation problem for $\mathrm{Hol}(\mathbb{D}, \Gamma)$ of analytic functions from $\mathbb{D}$ to Γ. We show by an example that a well-known necessary condition for the solvability of such an interpolation problem is not sufficient whenever the number of interpolation nodes is 3 or greater.

We introduce a sequence $\mathcal{C}_\nu, \nu \geq 0$, of necessary conditions for solvability of an n-point interpolation problem for $\mathrm{Hol}(\mathbb{D}, \Gamma)$ and show that they are of strictly increasing strength. Extremality in condition $\mathcal{C}_\nu$ leads us to define classes $\mathcal{E}_{\nu n}$ of rational Γ-inner functions, that is, analytic functions from $\mathrm{Hol}(\mathbb{D}, \Gamma)$ whose boundary values almost everywhere on the unit cirle lie in the distinguished boundary of Γ. The classes are related to n-extremality and provide a kind of classification of rational Γ-inner functions.

The talk is based on a joint work with Jim Agler and Nicholas Young.

[1] J. Agler, Z.A. Lykova and N.J. Young, Extremal holomorphic maps and the symmetrised bidisc, *Proceedings of the London Math. Soc.*, 26 October 2012, online doi: 10.1112/plms/pds049, 1-38.

Stefano Meda
Università di Milano-Bicocca

Hardy type spaces and harmonic Bergman spaces on the hyperbolic upper half-plane

Abstract. I shall define the Hardy-type space $X_1(D)$ and show that it plays in Harmonic Analysis on the hyperbolic disc much the same role as the classical real Hardy space $H_1(R)$ plays in Euclidean Harmonic Analysis. The space $X1_(D)$ is an atomic space. An atom in $X_1(D)$ is supported in a "small" geodesic ball B in D, and is orthogonal to all global harmonic functions on D, or, equivalently, to the harmonic Bergman space $b_2(B)$. This is joint work with G. Mauceri and M. Vallarino.

Mishko Mitkovski
Clemson University

On the reproducing kernel thesis for operators in Bergman-type spaces

Abstract. I will present several criteria that imply compactness of operators on a wide class of Hilbert function spaces. Most of these results can be viewed as reproducing kernel thesis statements. Namely, these results show that many crucial properties of a given operator can be deduced just by looking at its behavior on the reproducing kernels in the space. This talk is based on joint work with B. Wick.

Pavel Mozolyako
St Petersburg State University

Boundary behaviour of harmonic functions in growth spaces

Abstract. Consider the space h^∞_v of harmonic functions in $\mathbb{R}^{n+1}_+$ which satisfy the following growth restriction

$$|u(x,t)| \le Kv(t), \quad x \in \mathbb{R}^n,\ t \in \mathbb{R}_+.$$

We give description of the functions in this space in terms of their multiresolution approximation. When the weight grows faster than t^{-a} for some a, our result is in terms of the wavelet coefficients, for slow growing weights we consider partial sums of the wavelet series.
We then use this description to study the boundary behaviour of the functions in h^∞_v. In particular we obtain the following result about boundary oscillation of such functions

$$\limsup_{\delta\downarrow 0} \frac{\int_\delta^1 u(x,t)\, d\left(\frac{1}{v(t)}\right)}{\sqrt{\log v(\delta) \log\log\log v(\delta)}} \le C, \quad a.e.\, x \in \mathbb{R}^n,$$

for any $u \in h^\infty_v$ and any monotone v satisfying doubling condition.
This talk is based on a joint work with K. Eikrem and E. Malinnikova.

Artur Nicolau
Universitat Autònoma de Barcelona

Oscillation of Hölder continuous functions

Abstract. We study the local oscillation of a function in the euclidean space satisfying a Hölder condition and will show that its growth satisfies a version of the Law of the Iterated Logarithm. Joint work with Jose González Llorente.

Nikolai Nikolski
Université de Bordeaux 1, Steklov Inst. of Mathematics St Petersburg

Weakly invertible elements on the Hilbert multi-disc

Abstract. Weakly invertible (cyclic) elements of the Hardy space on an infinite dimensional disc D (included into l^2) are considered. We give a sufficient condition for the weak invertibility, discuss its possible sharpness and derive some consequences. In particular, the cyclicity of reproducing kernels is proved. An equivalent completeness problem for entire dilations, as well as its links to the Riemann conjecture, are considered too.

James Pascoe
University of California San Diego

Injective free polynomials

Abstract. We consider the injectivity of free polynomial maps evaluated on sets of matrices which are closed under similarity and direct sum. (For example, all matrices or commuting tuples of matrices.) We show such a map has an everywhere nonsingular derivative if and only if it is injective.

Jordi Pau
Universitat de Barcelona

Integration operators between Hardy spaces on the unit ball

Abstract. We completely describe the boundedness of the integration operator J_g from $H^p(B_n)$ to $H^q(B_n)$ extending the one dimensional results of Aleman-Cima-Siskakis.

Jose Angel Pelaez
University of Malaga

Trace ideal criteria for Toeplitz operators on weighted Dirichlet spaces

Abstract. Given a finite positive Borel measure μ on the unit disc $\mathbb{D}$ and a weighted Dirichlet space with reproducing kernels $K_z^\omega(\zeta)$, the Toeplitz operator T_μ is defined by

$$T_\mu f(z) = \int_{\mathbb{D}} f(\zeta)\, \overline{K_z^\omega(\zeta)}\, d\mu(\zeta).$$

If $\mathcal{D}_\alpha = \{f \in \mathcal{H}(\mathbb{D}) : \int_{\mathbb{D}} |f'(z)|^2 (1-|z|^2)^\alpha \, dA(z) < \infty\}$, a useful result of D. H. Luecking [2] describes the membership of T_μ to the Schatten class $\mathcal{S}_p(\mathcal{D}_\alpha)$ for all $p > 0$ with $p(1-\alpha) < 1$.

In this talk, we shall present a description of those measures μ for which the Toeplitz operator T_μ belongs to $\mathcal{S}_p(\mathcal{D}_\alpha)$ on the extended range of all $p > 0$ with $p(1-\alpha) < 2$ and $1 < p(2+\alpha)$.

We recall that a radial continuous weight ω satisfying

$$\lim_{r\to 1^-} \frac{\int_r^1 \omega(s)\, ds}{\omega(r)(1-r)} = \infty.$$

is called *rapidly increasing*. Every such weighted Bergman space A^p_ω lies between the classical Hardy space H^p and every standard Bergman space A^p_α.

It will also feature an extension of Luecking's result to a more general setting of weighted Dirichlet spaces induced by rapidly increasing weights.

Joint works with J. Pau and J. Rättyä.

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Karl-Mikael Perfekt
Lund University

Hankel forms and embedding theorems

Abstract. We discuss the boundedness of certain Hankel-type bilinear forms in the vector-valued setting, and its characterization in terms of Carleson embedding estimates. Based on joint work with Alexandru Aleman.

Stefanie Petermichl
Institut de Mathématiques de Toulouse

A characterization of product BMO.

Abstract. We consider commutators of multiplication by a BMO symbol and Calderon Zygmund operators. It is a classical result of Coifman, Rochberg and Weis which states that such commutators are always bounded if the symbol belongs to BMO (upper bound) and that the class of Riesz transforms is representative in the sense that boundedness of the commutators with all Riesz transforms implies that the symbol is in BMO (lower bound). Investigations by Uchiyama and Lee then replaced the collection of Riesz transforms by a much more general collection of CZOs. We extend this direction to the multi parameter situation where we face iterated commutators and product BMO. This is a generalization of a line of characterization results by Lacey and different collaborators. Joint work with Laurent Dalenc.

Alexei Poltoratski
Texas A&, College Station

Krein - de Branges theory and spectral problems

Abstract. The Krein - de Branges theory of Hilbert spaces of entire functions was created in 1940-60's to treat spectral problems for second order differential operators. It translates such problems into the language of complex and harmonic analysis, entire functions and singular integrals.

Since its creation the theory outgrew its original purpose. Multiple connections with other areas of analysis have been discovered. The latest additions to this list are connections to number theory and the Riemann Hypothesis that surfaced in the last 5-7 years.

In my talk I will discuss the basics of the theory and some of its newly found applications.

Sandra Pott
Lund University

Embedding Theorems in weighted Bergman spaces and applications to control theory

Abstract. We give an embedding theorem for a general class of weighted Bergman spaces on the half-plane with axis-parallel weights. The class of Bergman spaces is motivated by applications in control theory. This is joint work with Birgit Jacob (Wuppertal) and Jonathan Partington (Leeds).

Mihai Putinar
University of California, Santa Barbara - Nanyang Technichological University, Singapore

The spectrum of the Neumann-Poincaré operator on domains with corners

Abstract. The double layer potential, known also as the Neumann-Poincare operator, is the quintessential integral operator. Its spectrum has a physical meaning in potential theory and was studied for a century and a half. The talk will review some spectral bounds for planar domains with corners, starting with the thesis of Carleman and ending up with very recent numerical and theoretical discoveries. A multivariate analog of the N-P operator will also be discussed. Based on joint work with Karl-Mikael Perfekt.

Katie Quertermous
James Madison University

C*-algebras Generated by Composition Operators on Weighted Bergman Spaces

Abstract. If φ is an analytic self-map of the unit disk $\mathbb{D}$, then the composition operator $C_\varphi : f \mapsto f \circ \varphi$ is a bounded operator on the Hardy space $H^2(\mathbb{D})$ and on the weighted Bergman spaces $A^2_\alpha(\mathbb{D})$ for $\alpha > -1$. We are particularly interested in composition operators induced by linear-fractional self-maps of $\mathbb{D}$. Several authors have investigated the structures of C*-algebras generated by linear-fractionally-induced composition operators and either the unilateral shift or the ideal of compact operators on $H^2(\mathbb{D})$. In this talk, we extend some of these results to the weighted Bergman space setting by establishing a strong unitary equivalence between corresponding C*-algebras.

Fulvio Ricci
Scuola Normale Superiore, Pisa

Riemannian and subriemannian geometries on the Heisenberg group and operators generated by their combination

Abstract. Two natural geometries on the Heisenberg group are interesting for different reasons. One is the subriemannian geometry induced by its CR structure, the other is the left- and U(n)-invariant Riemannian geometry. It has been shown in recent years that both geometries coexist in several problems in complex, harmonic and functional analysis. This has generated an interest in algebras generated by operators proper of either geometry, like Riesz transforms and functions of (sub-)Laplacians.

Richard Rochberg[1]
Washington University in St. Louis

Analytic Structure in the Spectra of Multiplier Algebras of Dirichlet Type Spaces

Abstract. The Dirichlet type spaces are a family of reproducing kernel Hilbert spaces extending from the Hardy space to the Dirichlet space, The following result is classical for the algebra of bounded holomorphic functions, the multiplier algebra for the Hardy space: Every point in the spectrum which is in the closure of an interpolating sequence is the center of an analytic disk. I will discuss why the same is true for the other Dirichlet type spaces, except the endpoint case of the Dirichlet space itself. For the Dirichlet space itself, I can show that any such point is in a Gleason part of infinite cardinality, but I haven't been able to identify analytic disks.

William Ross
University of Richmond

Embeddings and reverse embeddings in de Branges-Rovnyak spaces

Abstract. In this talk I will present a survey of some recent joint work with Blandigneres, Fricain, Gaunard, and Hartmann about Carleson, reverse Carleson, and isometric embeddings of the deBranges-Rovnyak spaces. Particular focus will be on the dichotomy which takes place between the extreme and non-extreme cases.

Giulia Sarfatti
Università degli Studi di Firenze

Quaternionic Hardy Spaces

Abstract. The recent theory of slice regular functions over quaternions, introduced by Gentili and Struppa in 2006, presents several analogies with the theory of holomorphic functions of one complex variable. In this talk, after an introduction to the class of slice regular functions, I will discuss the construction of quaternionic Hardy spaces in this setting. Depending on the time, I will show some basic results for these function spaces, (joint work with Chiara De Fabritiis and Graziano Gentili).

[1]The speaker could not attend the conference.

Eric Sawyer
Mc Master University

Two weight singular integral inequalities in higher dimensions.

Abstract. We discuss two weight inequalities for singular integrals in Euclidean space, with an emphasis on functional energy. This is joint work with Chun-Yen Shen and Ignacio Uriarte-Tuero.

David Scheinker
Drexel University

A conjectured uniqueness condition for the Nevanlinna-Pick problem on the bidisc

Abstract. In this talk we give a geometric procedure for using Hilbert function spaces to construct a Nevanlinna-Pick problem on the bidisc with a specified set of uniqueness. Then, we conjecture a necessary and a sufficient condition for a Nevanlinna-Pick problem on the bidisc to have a unique solution and prove several special cases of this conjecture.

Carl Sundberg
University of Tennessee, Knoxville

Weak Product Spaces

Abstract. Let $\mathcal{H}$ be a Hilbert space of functions. We denote by $\mathcal{H} \odot \mathcal{H}$ the space of *weak products* of functions in $\mathcal{H}$, i.e. the set of functions h that can be written as $h = \sum f_n g_n$ for $f_n, g_n \in \mathcal{H}$ with $\sum \|f_n\|_{\mathcal{H}} \|g_n\|_{\mathcal{H}} < \infty$. Weak product spaces have appeared in a number of contexts in the literature, starting with the 1976 work of Coifman, Rochberg, and Weiss on factorizations in Hardy spaces of several variables.

We will discuss some technical issues in the definition of weak product spaces and then specialize to the case $\mathcal{H} = \mathcal{D}$, the Dirichlet space of analytic functions in the unit disk $\mathbb{D} \subset \mathbb{C}$ as well as its real analogue. Some results and open problems (more problems than results) will be discussed. This is joint work with Stefan Richter.

Attila Szalai
University of Szeged

Characterization of stability of contractions

Abstract. We characterize those sequences $\{h_n\}_{n=1}^{\infty}$ of bounded analytic functions, which have the property that an absolutely continuous contraction T is stable (that is the powers T^n converge to zero) exactly when the operators $h_n(T)$ converge to zero in the strong operator topology. Our result is extended to polynomially bounded operators too. The talk is based on a joint work with L. K'erchy.

Alexander Volberg
Michigan State University

A short proof of a nonhomogeneous $T1$ theorem. Application to Bergman spaces.

Abstract. Carleson measures for the family of Besov–Sobolev spaces in the unit ball are characterized by Brett Wick and the author. The main tool is a non-homogeneous $T1$ theorem, whose proof has always been quite involved. We will present a shorter proof, which is conceptually the most streamlined.

Brett Wick
Georgia Institute of Technology

Composition of Haar Paraproducts

Abstract. Paraproducts are important operators in harmonic analysis and there are well known characterizations of when an individual paraproduct is bounded. An interesting question is to characterize when the composition of two, potentially unbounded, paraproducts have a bounded composition. In this talk we will give necessary and sufficient conditions that characterize when the composition of certain compositions of Haar paraproducts are bounded.

Dmitry V. Yakubovich
Universidad Autonoma de Madrid

Dual Hilbert function spaces and dual functional models of linear operators

Abstract. As is know, Hilbert function spaces are closely related with functional models. One of the ways of obtaining a Hilbert function space is via reproducing kernels. Here we will discuss a different way, related to dual functional models of an operator and its adjoint. The models of an operator and of its adjoint are given in two different functional spaces, and there is a natural Cauchy duality between these spaces. In general, the model operator we obtain is only similar and not unitarily equivalent to the original one.

As it has been shown by the speaker, this approach works for different classes of operators, such as Toeplitz operators, hyponormal operators, non-dissipative perturbations of selfadjoint operators, generators of analytic semigroups and generators of semigroups related with delay and neutral linear systems.

After describing the general framework, we will speak on generators of analytic semigroups. Our main result is that a sectorial operator admits an H^∞-functional calculus if and only if it has a functional model of Nagy-Foiaş type. Furthermore, we give a simple concrete formula for the characteristic function (in a generalized sense) of such an operator. More generally, this approach applies to any sectorial operator by passing to a different norm (the McIntosh square function norm). We show that this norm is close to the original one, in the sense that there is only a logarithmic gap between them.

In this case, the dual functional models have the sense of an observation model and a control model and can be defined by basic linear control theory.

If time permits, we will also speak on models of a subnormal operator and more general models and their relation with separated algebraic curves in $\mathbb{C}^2$ (a joint work in progress with D. Estévez).

Nicholas Young
Leeds University, Newcastle University

Realization of symmetric analytic nc-functions

Abstract. We study symmetric analytic nc-functions on the *biball*

$$B^2 \stackrel{\text{def}}{=} \bigcup_{n=1}^{\infty} \mathbb{B}_n \times \mathbb{B}_n,$$

where $\mathbb{B}_n$ denotes the open unit ball of the space $\mathcal{M}_n$ of $n \times n$ complex matrices. B^2 is the non-commutative analogue of the bidisc. We show that every such function that is bounded by 1 in norm factors through a certain nc-domain Ω in the space

$$\mathcal{M}^\infty \stackrel{\text{def}}{=} \bigcup_{n=1}^{\infty} \mathcal{M}_n^\infty.$$

Here an *nc-function* is a function defined on an nc-domain Ω in $\bigcup_{n=1}^\infty \mathcal{M}_n^d$ (for some $d \leq \infty$) that respects direct sums and similarities and maps $\Omega \cap \mathcal{M}_n^d$ to $\mathcal{M}_n$. An *nc-domain* is a domain in $\bigcup_{n=1}^\infty \mathcal{M}_n^d$ that is closed under direct sums and unitary similarity.

More precisely, there exists an nc-domain Ω in $\mathcal{M}^\infty$ and an analytic nc-function $S : B^2 \to \Omega$ (given by a simple rational expression) with the following property. For every symmetric nc-function φ on B^2 that is bounded by 1 in norm there exists an analytic nc-function Φ on Ω such that $\varphi = \Phi \circ S$; moreover Φ can be expressed by means of a non-commutative version of the familiar linear fractional realization formula for functions in the Schur class.